The Effect of Activists’ Short-Termism on Corporate Governance

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Abstract

Does activist investors’ focus on short-term stock prices impede their role in improving corporate governance? This model builds on the notion that both the act of intervention and the threat of an intervention can generate value for the target firm. We demonstrate that an activist with short-termism intervenes excessively (insufficiently) when her ability to conduct a value-enhancing intervention is low (high). However, the excessive intervention from a less capable activist can be value-enhancing, because of the disciplining effect of shareholder activism on managerial incentives. Moreover, we show that an activist with higher ability can mitigate her lack of incentives to intervene by acquiring stakes in multiple firms even when her capacity to conduct intervention is limited. Because targeting multiple firms dilutes the effect of the activist’s threat on each manager, the skilful activist conducts more value-enhancing intervention in equilibrium. Finally, when facing an activist with low ability, short-term stock-based managerial compensation disincentivizes effort. Increasing activist short-termism reduces such perverse effect.
1 Introduction

Shareholder activism has long been recognized as an important corporate governance mechanism. Activist shareholders can add value by intervening in a firm’s operations and implementing value-enhancing strategies (e.g., Shleifer and Vishny 1986; Admati, Pfleiderer, and Zechner 1994). However, the positive effect of shareholder activism on corporate governance goes beyond direct interventions: managers facing the threat of a potential intervention take preemptive measures to improve their firm’s performance in an attempt to avoid an intervention (Gantchev, Gredil, and Jotikasthira 2015).

Despite these positive effects of shareholder activism, industry experts have voiced concerns about the relatively short horizon of activist investors. In a speech given in June 2015, SEC commissioner Daniel Gallagher notes that “there seems to be a predominance of short-term thinking at the expense of long-term investing. Some activists are swooping in, making a lot of noise, and demanding one of a number of ways to drive a short-term pop in value: spinning off a profitable division, beginning a share buy-back program, or slashing capital expenditures or research and development expenses” (Gallagher 2015). The empirical analysis by Bebchuk, Brav, and Jiang (2015) also centers around the claim that “myopic” activists may harm long-term corporate value.

In this paper, we develop a dynamic rational expectations model to analyze an activist shareholder’s optimal investment and intervention policy under short-termism, and study the effects on corporate governance. Our model incorporates three critical elements of shareholder activism: (i) the activist’s focus on short-term stock prices, (ii) other market participants’ limited information about the activist’s ability to implement value-enhancing strategies, and (iii) the manager’s ability to proactively take actions to lower the benefits of an activist intervention. While we show that short-termism generally leads the activist to adopt an inefficient intervention policy, the effect on corporate value is not always negative, depending on the activist’s expected ability and the manager’s incentive structure.

Our analysis starts with the notion that both the act of intervention and the threat of an inter-

1 Alternatively, large shareholders can discipline management via the threat of exit, also known as the “Wall Street Walk” (e.g., Admati and Pfleiderer 2009; Edmans 2009; Edmans and Mansol 2011; Goldman and Strobl 2013; Dasgupta and Piacentino 2014).

2 The authors find no support for this claim in the data on average, a result that is consistent with predictions of our model. Our model provides further testable predictions regarding the effect of activists’ short-termism in the cross section of firms and activists.
vention can generate value for the target firm. We focus on an activist investor who can implement an alternative strategy in the target firm. While the true value of the activist’s alternative strategy is not observable by other market participants, the market knows the probability that the strategy is value-enhancing. We refer to this as the activist’s ability. If the activist intervenes and implements her alternative strategy, the manager incurs a private cost. This cost can be interpreted as the manager’s disutility from getting fired, or simply as a reduction in the manager’s private benefits associated with a loss of control. The threat of intervention thus creates an incentive for the manager to proactively exert costly effort to improve firm value, thereby reducing the likelihood of an activist intervention. Depending on whether there is an intervention or not, the value of the firm depends on the value of the activist’s alternative strategy or the outcome of the manager’s effort choice, respectively.

While the activist’s objective is to maximise the profit upon exiting her position in a target firm, the horizon of the activist is related to the extent of information asymmetry between the activist and other market participants at the time of exit. That is, the true value of the firm remains private information at the time of exit with greater probability, if the activist has greater short-termist concerns. When the true value of the firm indeed remains private information upon activist exit, the market price of the firm depends on the expected value of the activist’s alternative strategy. The information advantage of the activist allows her to intervene strategically in an attempt to manipulate the market price of the firm upon her exit.

Our first result shows that, depending on her ability, the activist’s optimal intervention strategy can entail too much or too little intervention (relative to the first-best level). An activist with low ability intervenes excessively, even when her alternative strategy is value-destroying, whereas a capable activist sometimes foregoes value-enhancing interventions. Furthermore, the range of ability level for which the activist intervenes efficiently (at the first-best level) diminishes with increasing short-termism.

The intuition for this result is as follows. When the activist has a stronger short-termism, she focuses less on the fundamental value of her intervention. Instead, her intervention strategy trades off the market’s perceived value of her intervention against the value of the manager’s expected effort.

\[^3\] For example, Brav, Jiang, Partnoy, and Thomas (2008) show that CEO pay drops by $1 million and CEO turnover goes up by 10% in the year following a hedge fund activist intervention.
induced by the threat of activism. While excessive intervention reduces the market’s perceived value of an intervention, insufficient intervention decreases the credibility of the intervention threat. The incentive to intervene excessively dominates for an activist with low ability, because the manager exerts little effort when facing an activist with low ability, anticipating her inability to implement a profitable strategy. In contrast, a highly capable activist relies predominantly on the threat of intervening and sometimes foregoes a profitable intervention opportunity.

While short-termism generally leads an activist to adopt inefficient intervention strategies, the overall effect on corporate governance is not always negative. Consider a low-ability activist. Because intervene excessively, she imposes a stronger threat on the manager in equilibrium than if she were to intervene efficiently. This can benefit the firm value, if the manager is readily motivated and can exert productive effort. If this is the case, the value of increased managerial effort outweighs the deadweight loss of value destroying intervention by the activist. Otherwise, activists’ short-termism indeed impedes corporate governance. In particular, a highly capable activist foregoes profitable intervention opportunities. This not only has a direct effect on the firm value, but also decreases the incentive for the manager to exert effort, further reducing the firm value.

We next consider the activist’s incentive to target multiple firms simultaneously, and show that the a highly capable activist can alleviate the problem of insufficient intervention by optimally acquiring stakes in multiple firms, even if she is capacity constrained and can only intervene in one firm. Intuitively, there are costs and benefits associated with targeting multiple firms. Since the manager of each target firm exerts effort under the threat of activism, the activist profits from investing in additional firms even if she does not necessarily intervene in these firms. However, having multiple targets stretches the activist too thinly and dilutes the threat of activism for each firm, leading to a lower level of effort by each manager.

The intuition for the optimality of multiple targets hinges on the interplay between the manager’s effort provision and the activist’s incentive to intervene. Under short-termism, a decrease in the expected effort provision of the manager increases the activist’s intervention frequency in equilibrium, because lower managerial effort reduces the payoff the activist receives if she does not intervene. Since a highly reputable activist under short-termism tends to intervene insufficiently, having multiple targets dilutes the threat imposed by the activist on each manager, allowing the activist to conduct more value-enhancing interventions in equilibrium. For an activist with low
ability, on the other hand, focusing on one target is optimal. Since she tends to intervene excessively, the dilution effect of multiple targeting on the manager’s effort choice further decreases the efficiency of her interventions, as she conducts even more value-destroying interventions.

Finally we investigate how activist shareholders’ short-termism interacts with manager short-termism, when the manager receives stock-based compensation. Long-term stock-based compensation depends on the fundamental value of the firm, while short-term stock-based compensation is sensitive to the share price changes induced by activist intervention. Surprisingly, we find that activists’ short-termism can improve the incentive effect of managerial compensation. We show that, while long-term incentives are efficient in inducing effort by the manager, its effectiveness is decreasing in the activist shareholders’ horizon. This is because increasing activist shareholders’ horizon improves the efficiency of her intervention after manager failure. This increases the payoff the manager receives following a failure, reducing his incentive to exert effort.

Furthermore, when facing an activist with low ability, short-term stock-based compensation has a disincentivizing effect on the manager, which can be mitigated by increasing activist short-termism. Recall the earlier result that, due to the limited threat of activism from a low-ability activist, the manager exerts little effort and the market perceived value of the firm in the absence of an intervention is relatively low. Since an intervention from an activist with low ability leads to a higher share price, the manager receiving short-term stock-based compensation prefers to exert less effort, in order to induce intervention from the activist. In this case, increasing activist shareholders’ short-termism mitigates such perverse effect, as the activist’s strategic intervention attempting to profit from the same short-term price discrepancy reduces the price increase following an intervention in equilibrium.

1.1 Related Literature

Our paper is related to several strands of the literature. First, it is related to the literature on how shareholder intervention can improve firm value. The existing literature is based on the premise that shareholder interventions increase firm value ex post at a private cost (e.g., Shleifer and Vishny, 1986; Kyle and Vila, 1991; Admati, Pfleiderer, and Zechner, 1994; Kahn and Winton, 1998; Maug, 1998). In this paper, we consider an activist shareholder who can engage in intervention at no cost, but who has private information regarding the value of her action, which can be ex post value-
enhancing or value-destroying. The model thus focuses on a pure informational friction between the activist and the market, interpreted as short-termism, instead of an agency problem as in existing literature. This leads to implications for the probability of ex post value-destroying interventions in equilibrium.

Bebchuk, Brav, and Jiang (2015) investigate empirically whether interventions by activist shareholders have an adverse effect on the long-term interests of companies due to activists’ short-termism. Consistent with our model, they found that activist intervention are followed by improved operating performance in the subsequent five-year period. Our model produces additional empirical predictions in the cross section regarding the effect of activist intervention depending on the perceived ability of the activist, and the managerial compensation structure of the firm.

Our paper also contributes to the literature that analyzes the effect of shareholder interventions on managers’ incentives (e.g., Grossman and Hart 1980, Scharfstein 1988). The ex ante threat of intervention thus interacts with the activist’s strategy to conduct an intervention ex post. While the before-mentioned papers show that takeover threats have a disciplining effect, they also create some potential conflicts. Stein (1988) argues that takeover pressure can be damaging because it encourages managers to sacrifice long-term interests in order to boost current profit. Burkart, Gromb, and Panunzi (1997) show that tight shareholder control is ex post efficient, but may ex ante discourage managerial initiative. Our paper contributes to this literature by showing that the activist’s ex post intervention decision and the effect of the threat of such an intervention on the manager’s ex ante effort choice can be complements or substitutes.

Finally, our paper is related to the literature on investor’s portfolio concentration. Focusing on the threat of exit, Edmans, Levit, and Reilly (2015) show that a multi-firm structure can improve governance because the exit of one firm while retaining another is a powerful signal of underperformance. Inderst, Mueller, and Münnic (2007) and Fulghieri and Sevilir (2009) show that the competition between portfolio firms can encourage managers to exert greater effort. Our paper points to a different mechanism through which targeting multiple firms adds value even when the activist has limited intervention capacity. In contrast to existing literature, our mechanism relies on the dilution effect of a larger portfolio on the manager’s effort. The activist adds value in equilibrium as she is forced to substitute the reduced manager effort with more value-enhancing intervention.
The rest of the paper is organized as follows. Section 2 describes the model setup. Section 3 characterizes the equilibrium when the activist targets one firm, and analysis the effect of the activist’s reputation on her optimal intervention strategy, the target firm manager’s effort, and the firm value in equilibrium. Section 4 then investigates the trade off faced by the activist when targeting multiple firms. Finally Section 6 concludes.

2 The Model

We consider an economy with a single firm and three types of risk-neutral agents: an incumbent firm manager (M), an activist shareholder (A), and competitive investors. The game has four dates, denoted by \( t \in \{0, 1, 2, 3\} \). The riskless rate is normalized to zero.

At \( t = 0 \), the activist shareholder obtains a block of the firm’s shares, normalized to one unit, that gives her sufficient voting rights to intervene in the firm’s operations (for example, by replacing the incumbent management or by winning board seats in director elections). As will become clear below, the size of the activist’s stake is inconsequential for our analysis, as long as it allows the activist to implement her preferred policy.

We further assume that activist is able to obtain the block secretly at price \( v \), which corresponds to the value of the firm in the absence of an activist shareholder. That is, we assume that the activist captures all the value creation to the target firms due to her presence. This assumption is without loss of generality. The results still go through if part of the value is reflected in the acquisition price and the activist only captures a given portion of the value created.

In the absence of an intervention by the activist, the firm’s (per unit) payoff is \( v^M \in \{v^M_h, v^M_\ell\} \), where \( v^M_\ell < v^M_h \). At \( t = 1 \), the manager can improve firm value through exerting unobservable effort \( e \in [0, 1] \). In particular, by incurring a utility cost of \( ce^2/2 \), he can increase the firm’s payoff from \( v^M_\ell \) to \( v^M_h \) with probability \( e \).

At \( t = 2 \), the activist shareholder privately observes the firm’s payoff \( v^M \) generated by the incumbent manager (in the absence of an intervention by the activist). At the same time, the activist also observes the potential payoff \( v^A \in \{v^A_h, v^A_\ell\} \), \( v^A_\ell < v^A_h \), that the firm could achieve.

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4In Section 4, we extend the model to a multiple-firm setting in which the activist can potentially target two firms simultaneously, to study the activist’s optimal choice of number of targets.

5In the remainder of this article, we refer to the activist shareholder as a woman, while we take the firm manager to be a man.
if she were to intervene in the firm’s operations and to implement an alternative strategy. Based on this information, the activist then decides whether or not to intervene in the firm’s operations (denoted by $I = 1$ and $I = 0$, respectively). An intervention by the activist has two effects. First, it imposes a utility cost of $K$ on the incumbent manager. This cost can be interpreted as the manager’s disutility from getting fired, or simply as a reduction in the manager’s private benefits associated with a loss of control. Second, an intervention changes the firm’s payoff from $v^M$ to $v^A$, the payoff generated by the activist’s alternative strategy. For simplicity, we assume that the activist can choose to intervene only if the incumbent manager is unsuccessful (i.e., if $v^M = v^M_I$)\(^6\)

The activist’s intervention decision $I$ is observable by all agents. The payoff $v^A$ that the activist’s alternative strategy generates depends on the activist’s ability, $\mu \in (0, 1)$. An intervention by an activist of ability $\mu$ leads to a high payoff of $v^A_h$ with probability $\mu$. With probability $1 - \mu$, the payoff of the activist’s alternative proposal is $v^A = v^A_I$. The activist’s ability is unknown to all agents, including the activist herself. We make the following assumption about the firm’s payoffs.

**Assumption 1.** $0 \leq v^A_I < v^M_I < v^A_h < v^M_h$.

This assumption reflects the following empirical observations. An activist shareholder can potentially improve firm value if the manager fails to perform ($v^A_h > v^M_I$). However, due to his expertise and knowledge about the firm, a skilled manager can do better than an outside activist shareholder can ($v^M_h > v^A_I$). Further, a misguided activist can do more harm to the firm than a complacent manager ($v^M_I > v^A_h$).

At $t = 3$, the activist exits by selling her shares to investors. Investors are assumed to be perfectly competitive. Thus, the price $P$ that investors are willing to pay for the activist’s stake equals the firm’s expected payoff, conditional on the investors’ information. With probability $\gamma$, the firm’s payoff is publicly revealed before the market opens at $t = 3$. If this is the case, the price equals either $v^A$ (if the activist intervenes) or $v^M$ (if the activist does not intervene). With probability $1 - \gamma$, the firm’s payoff does not become public before the activist sells her shares. In this case, the price only depends on the activist’s intervention decision. We denote this price by $P_I$, $I \in \{0, 1\}$. $\gamma$ thus measures the degree of information asymmetry in the market, for the relevant

\(^6\)Under Assumption 1, it can be shown that an equilibrium in which the activist can intervene also when the manager is successful is less efficient than if she cannot.
\[ t = 0: \quad \bullet \ A \text{ obtains a stake in the firm} \]
\[ t = 1: \quad \bullet \ M \text{ chooses unobservable effort } e \]
\[ t = 2: \quad \bullet \ A \text{ privately observes } v^M \text{ and } v^A \\
\quad \bullet \text{ If } v^M = v^M_L, \text{ } A \text{ decides whether or not to intervene} \]
\[ t = 3: \quad \bullet \text{ Firm’s payoff is publicly revealed with probability } \gamma \\
\quad \bullet \ A \text{ sells her stake to investors at price } P \]

Table 1: Sequence of events.

The activist shareholder chooses her intervention strategy to maximize the market value of her equity stake at \( t = 3 \) (that is, the price \( P \) at which she sells her shares). The manager minimizes her total cost, which consists of two components, her cost of effort, \( ce^2/2 \), and the utility cost she incurs if the activist intervenes, \( K \). We make the following assumption to ensure that the equilibrium effort level satisfies \( e \in [0, 1] \).

**Assumption 2.** \( \frac{K}{e} \leq 1 \).

To highlight the incentive effect of the activist’s intervention on the manager’s effort choice, we abstract from explicit monetary incentives. However, our analysis in Section xxx shows that, under threat of shareholder activism, stock-based managerial compensation contracts has no incentivizing effect when information asymmetry in the market is high (\( \gamma \to 0 \)), and can disincentivize the manager when faced with an activist with high prior reputation (\( \mu \)).

The sequence of events in the model is summarized in Table 1.

### 3 Shareholder Activism under Short-Termism

In this section we study the activist’s optimal intervention strategy and the effect of her intervention strategy on the manager’s effort. We first establish a benchmark where the activist is absent, then proceed to analyze the equilibrium and discuss the effect of the short-termism of the activist investors.
3.1 Benchmark without an Activist

As a benchmark, we characterize the effort level of the manager and the value of the firm. This is then taken as the price at which the activist can acquire the block of shares in the target firm.

We start by considering the manager’s effort choice. In the absence of an activist shareholder, the manager chooses zero effort to minimize only his effort cost $\frac{e^2}{2}$. As a result, the expected value of the firm in the absence of an activist shareholder is simply $v^M_\ell$. This benchmark result is summarized in the following lemma.

**Lemma 1.** In the absence of an activist shareholder, the manager exerts zero effort and the value of the firm is given by $v = v^M_\ell$.

3.2 Equilibrium and the Effect of Short-Termism

In this section, we solve for the equilibrium intervention policy of the activist, which is fully characterised by $\{\lambda_h, \lambda_\ell\}$, which denote the probability of intervention given the activist’s outcome $v_A^h$ and $v_A^\ell$ respectively. We solve the model starting from $t = 3$ and working backwards.

At $t = 3$, with probability $(1 - \gamma)$ no additional information is revealed. The market price given an activist’s intervention strategy and the manager’s effort level $e$ is as follows:

$$P_1 \equiv \mathbb{E}[v \mid I = 1] = Pr[v^A = v^A_h \mid I = 1](v^A_h - v^A_\ell) + v^A_\ell$$

$$= \frac{\mu \lambda_h}{\mu \lambda_h + (1 - \mu) \lambda_\ell}(v^A_h - v^A_\ell) + v^A_\ell$$

$$P_0 \equiv \mathbb{E}[v \mid I = 0] = Pr[v^M = v^M_h \mid I = 0](v^M_h - v^M_\ell) + v^M_\ell$$

$$= \frac{e}{e + (1 - e)[\mu(1 - \lambda_h) + (1 - \mu)(1 - \lambda_\ell)]}(v^M_h - v^M_\ell) + v^M_\ell$$

Notice that the expected firm value following an intervention depends only on the activist’s strategy, whereas that following no intervention depends also on the manager’s effort.

At $t = 2$, if the manager fails ($v^M = v^M_\ell$), the activist decides upon her intervention strategy, given her private information $v^A \in \{v^A_h, v^A_\ell\}$, to maximise her expected payoff at exit, taking the
market prices $P_I, I \in \{1, 0\}$ as given:

$$\lambda_h \equiv \arg \max_\lambda \left[ \lambda \gamma v^A_h + (1 - \gamma) P_1 + (1 - \lambda) \gamma v^M_h + (1 - \gamma) P_0 \right]$$

$$\lambda_\ell \equiv \arg \max_\lambda \left[ \lambda \gamma v^A_\ell + (1 - \gamma) P_1 + (1 - \lambda) \gamma v^M_\ell + (1 - \gamma) P_0 \right]$$

Finally at $t = 1$, the manager chooses an effort level $e$ in anticipation of the activist’s intervention strategy to minimize his costs, including both the effort cost and the expected cost induced by activist intervention:

$$\min_e \frac{ce^2}{2} + K(1 - e) \Lambda(\mu, \lambda_h, \lambda_\ell) \quad (4)$$

where

$$\Lambda(\mu, \lambda_h, \lambda_\ell) \equiv Pr[I = 1 | v^M = v^M_\ell] = \mu \lambda_h + (1 - \mu) \lambda_\ell \quad (5)$$

$\Lambda(\mu, \lambda_h, \lambda_\ell)$ is the conditional expected intervention probability in case the manager fails. It follows that the optimal effort choice of the manager is $e(\Lambda(\mu, \lambda_h, \lambda_\ell)) = \frac{K}{c} \Lambda(\mu, \lambda_h, \lambda_\ell) = k \Lambda(\cdot)$, where $k \equiv \frac{K}{c} \leq 1$ denotes the normalised cost of intervention to the manager.

Having now characterised the activist’s and the manager’s problems, we now solve for the perfect Bayesian equilibrium (PBE) in this model. Formally, a PBE consists of the manager’s effort choice, the activist’s intervention strategy, and a system of beliefs such that (1) the choices made by the manager and the activist maximise their respective objective functions, given the equilibrium choices of the other agent and the equilibrium beliefs, (2) the beliefs are rational given the equilibrium choices of the agents and are formed using Bayes’ rule (whenever possible). We hence forth denote by * all equilibrium quantities.

**Proposition 1.** There exist thresholds $\underline{\mu}$ and $\overline{\mu}, \underline{\mu} < \overline{\mu}$, such that the unique equilibrium is as follows:

- For $\mu < \underline{\mu}$,
  - A’s intervention strategy is $\lambda^*_h(\mu) = 1$ and $\lambda^*_\ell(\mu) \in (0, 1)$;
  - M’s effort level is $e^*(\mu) = k[\mu + (1 - \mu)\lambda^*_\ell(\mu)]$;
- Equilibrium prices are

\[ P_1^*(\mu) = \frac{\mu}{\mu + (1 - \mu)\lambda^*_{h}(\mu)} (v^A_h - v^A_{\ell}) + v^A_{\ell} \]

\[ P_0^*(\mu) = \frac{e^*(\mu)}{e^*(\mu) + [1 - e^*(\mu)](1 - \mu)(1 - \lambda^*_{h}(\mu))} (v^M_{h} - v^M_{\ell}) + v^M_{\ell} \]  

(6) (7)

- \( \lambda^*_{h}(\mu) \) is given by \( P_1^*(\cdot) - P_0^*(\cdot) = \frac{\gamma}{1 - \gamma} (v^M_{h} - v^M_{\ell}) \).

- For \( \mu \in [\mu, \bar{\mu}] \),

- A’s intervention strategy is \( \lambda^*_{h}(\mu) = 1 \) and \( \lambda^*_{\ell}(\mu) = 0 \);
- M’s effort level is \( e^*(\mu) = k\mu \);
- Equilibrium prices are

\[ P_1^*(\mu) = v^A_h \]  

\[ P_0^*(\mu) = \frac{e^*(\mu)}{e^*(\mu) + [1 - e^*(\mu)](1 - \mu)} (v^M_{h} - v^M_{\ell}) + v^M_{\ell} \]  

(8) (9)

- For \( \mu > \bar{\mu} \),

- A’s intervention strategy is \( \lambda^*_{h}(\mu) \in (0, 1) \), \( \lambda^*_{\ell}(\mu) = 0 \);
- M’s effort level is \( e^*(\mu) = k\mu\lambda^*_{h}(\mu) \);
- Equilibrium prices are

\[ P_1^*(\mu) = v^A_h \]  

\[ P_0^*(\mu) = \frac{e^*(\mu)}{e^*(\mu) + [1 - e^*(\mu)](1 - \mu)} (v^M_{h} - v^M_{\ell}) + v^M_{\ell} \]  

(10) (11)

- \( \lambda^*_{h}(\mu) \) is given by \( P_1^*(\cdot) - P_0^*(\cdot) = \frac{\gamma}{1 - \gamma} (v^A_h - v^M_{\ell}) \)

- \( \mu \) is given by \( v^A_h - P_0'(\mu) = \frac{\gamma}{1 - \gamma} (v^M_{\ell} - v^A_{\ell}) \), and \( \bar{\mu} \) is given by \( P_0'(\mu) - v^A_h = \frac{\gamma}{1 - \gamma} (v^A_h - v^M_{\ell}) \), where \( P_0'(\mu) \) is given by Eq. 2 for \( \lambda_{h} = 1 \), \( \lambda_{\ell} = 0 \) and \( e = k\Lambda(\mu, 1, 0) \).

All proofs are in Appendix 7. Here we discuss the intuition behind our result.
The intervention strategy of the activist in equilibrium, depends on the market prices at which she can exit her position. The solution to the activist’s maximisation problem (Eq. 3) is follows:

\[
\lambda = \begin{cases} 
1, & \text{if } \gamma(P_1 - P_0) > (1 - \gamma)(v^M - v^A) \\
\in [0, 1], & \text{if } \gamma(P_1 - P_0) = (1 - \gamma)(v^M - v^A), \quad \forall v^A \in \{v^A_h, v^A_l\} \\
0, & \text{if } \gamma(P_1 - P_0) < (1 - \gamma)(v^M - v^A)
\end{cases}
\] (12)

On the one hand, there is a chance that the true value of the firm is revealed to the market, so she cares about the effect of her intervention decision on the firm value \((v^M - v^A)\). Ex post efficient intervention should occur if and only if \(v^A = v^A_h\), as \(v^A_h > v^M > v^A_l\). On the other hand, if the true value is not revealed, the activist has the incentive to intervene if it leads to a higher price, even if intervention destroys firm value. Similarly, the activist may forego an efficient intervention opportunity, if intervention is associated with a lower price.

In turn, the intervention strategy of the activist also affects the market prices in equilibrium. Intervention increases \(P_0\) because of two re-enforcing effects. First, intervention increases the cost to the manager in case he fails, resulting in higher manager effort ex ante and hence the value of the firm even in the absence of a direct intervention. Secondly, more intervention in case of management failure increases the posterior probability of management success given the absence of a direct intervention. A higher \(P_0\), however, reduces the activist’s incentive to intervene. The effect of the activist’s intervention on \(P_1\) depends on the activist’s alternative proposal \(v^A\). Inefficient intervention, i.e. a lower \(\lambda_h\) or a higher \(\lambda_l\), decreases \(P_1\) because \(P_1\) reflects the expected value of the activist’s intervention.

Proposition 1 shows that, which of the above forces dominates in equilibrium depends on the activist’s ability \(\mu\). An activist with low ability \(\mu < \mu^*\) intervenes excessively in equilibrium. Faced with an activist with low reputation, the manager effort level is low and the firm value in the absence of intervention \(P_0\) is low. This creates an incentive for the activist with \(v^A = v^A_l\) to intervene with positive probability, so as to pool with the activists with \(v^A = v^A_h\) and receive a higher price \(P_1\), even when her intervention destroys firm value.

For an activist with high ability \(\mu \geq \mu^*\), the opposite is the case. The price is high in the absence of activist intervention because the manager exerts high effort, anticipating that the activist receives
\( v^A = v^A_h \) with high probability and therefore intervenes often. This then allows the activist to bank on her reputation in case the manager fails \( v^M = v^M_\ell \), and forego efficient intervention when \( v^A = v^A_h \) with positive probability, so as to pool with the a firm with \( v^M = v^M_h \) and receive a higher price \( P_0 \).

Short-termism thereby introduces a two-sided inefficiency, as illustrated in Figure 1. The dashed line represents the ex post efficient intervention strategy \((\lambda_h, \lambda_\ell) = (1, 0)\), where the activist intervenes if and only if she has a value-enhancing investment opportunity \( v^A_h \). The thick solid line represents the equilibrium intervention strategy of the activist. Compared to the ex post efficiency strategy, the activist driven by reputation concern intervenes excessively if her reputation is low \( \mu < \mu_0 \), but intervenes insufficiently if her reputation is high \( \mu > \mu_0 \).

Figure 1: Equilibrium intervention strategy

The following corollary examines more closely the equilibrium effect of an increase in the activist’s ability. Denote with \( \Lambda^*(\mu) \equiv \Lambda(\mu, \lambda^*_h(\mu), \lambda^*_\ell(\mu)) = \mu \lambda^*_h(\mu) + (1 - \mu) \lambda^*_\ell(\mu) \) the expected intervention probability conditional on management failure in equilibrium.

**Corollary 1.**

- A’s intervention strategy \( \lambda^*_h(\mu) \) is equal to 1 for \( \mu \leq \mu_0 \) and strictly decreasing for \( \mu > \mu_0 \), whereas \( \lambda^*_\ell(\mu) \) is strictly positive for \( \mu < \mu_0 \), and equal to 0 for \( \mu \geq \mu_0 \).

- The expected intervention probability conditional on management failure \( \Lambda^*(\mu) \) and hence M’s effort level \( e^*(\mu) \) are strictly increasing for \( \mu < \mu_0 \), and equal to \( \Lambda^*(\mu_0) \) and \( e^*(\mu_0) \), respectively, for \( \mu \geq \mu_0 \).
The equilibrium prices $P_1^*(\mu)$ and $P_0^*(\mu)$ are increasing in $\mu$. $P_1^*(\mu)$ is strictly increasing for $\mu < \underline{\mu}$ and equal to $P_1^*(\mu)$ for $\mu \geq \underline{\mu}$. $P_0^*(\mu)$ is strictly increasing for $\mu < \bar{\mu}$ and equal to $P_0^*(\bar{\mu})$ for $\mu \geq \bar{\mu}$.

This corollary is illustrated in Figure 2. In the left panel, the solid line represents the equilibrium effort level of the manager $e^*(\mu)$, whereas the dotted line represents the effort level of the manager under first best intervention strategy $(\lambda_h, \lambda_\ell) = (1, 0)$. The right panel plots the equilibrium prices $P_1^*(\mu)$ and $P_0^*(\mu)$.

Figure 2: Equilibrium manager effort and firm prices

Corollary 1 indicates an interesting property of the equilibrium. The two mechanisms through which activism creates value, namely manager effort provision and the activist’s intervention, can be complements or substitutes depending on the activist’s ability. For $\mu \leq \underline{\mu}$, they are complements. That is, an increase in the activist’s ability improves both the manager’s effort and the efficiency of the activist’s intervention strategy, as she reduces the probability of ex post inefficient intervention $\lambda^*_\ell(\cdot)$. For $\mu \geq \bar{\mu}$, however, these are substitutes. That is, an increase in the activist’s ability improves the manager’s effort, but reduces the efficiency of the activist’s intervention strategy, as she reduces the probability of ex post efficient intervention $\lambda^*_h(\cdot)$. In Section 4 we further explore the consequences of this result in the activist’s incentive to target multiple firms simultaneously.

Furthermore, the positive effect of an increase in the activist’s ability is diminishing in equilibrium. Although in equilibrium the manager still faces an increasing threat of intervention from
a more capable activist, the incentives for the activist to intervene ex post, $\lambda_h^*(\mu)$ and $\lambda^*_l(\mu)$, eventually diminishes as the activist’s ability exceeds $\bar{\mu}$. This is driven by the activist’s incentive to strategically choose actions that lead to a higher price at exit, when the observability of the true effect on the firm value is limited. This can be seen in Fig. 1 and 2 where the equilibrium intervention probability $\Lambda^*(\mu)$, manager effort $e^*(\mu)$ and prices $P^*_0(\mu)$ are constant for $\mu \geq \bar{\mu}$.

**Lemma 2.** The thresholds are such that $\bar{\mu}$ is strictly increasing in $\gamma$, and $\mu$ is strictly decreasing in $\gamma$. In particular,

- As $\gamma \to 1$, $\bar{\mu} \to 1$ and $\mu \to 0$.
- As $\gamma \to 0$, $\bar{\mu} \to \mu$, and $P^*_0(\mu) \to P^*_1(\mu)$.

This lemma highlights the effect of the activist shareholder’s short-termism. An activist shareholder with long horizon $\gamma \to 1$ always implements the first-best intervention strategy. As the horizon of the activist shareholder shortens, the region of ability associated with efficient intervention, $[\mu, \bar{\mu}]$ shrinks and an region with excessive intervention emerges for $\mu \in (0, \mu)$. In the limit where no additional information regarding the firms fundamental is revealed at the time of exit of the activist, $\gamma \to 0$, the equilibrium entails either too much or too little intervention, relative to the first-best level.

For the ease of exposition we focus on the limiting case as $\gamma \to 0$ in the rest of the analysis. In this case, the equilibrium is characterised by a single threshold level of ability $\mu$. Denote this threshold $\mu^*$. The following proposition examines how the firm characteristics $v^M$ and the manager’s cost of effort $K$ and $c$ affects the activist’s intervention strategy and the expected firm value.

**Proposition 2.** As $\gamma \to 0$,

- The expected intervention probability conditional on management failure $\Lambda^*(\mu)$ is strictly decreasing in $v^M_h$, $v^M_l$, and $k$.
- The manager’s equilibrium effort level $e^*(\mu)$ is strictly decreasing in $v^M_h$ and $v^M_l$, and strictly increasing in $k$.

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7The limit serves as an equilibrium selection device for when the market is completely uninformed about the realisation of the true firm value. The limiting case eliminates the undesirable equilibria in which $\lambda_h < 1$ and $\lambda_l > 0$, because the activist with $v^A = v^A_h$ and $v^A = v^A_l$ face the same incentives when she does not internalise any of the direct consequence of her intervention action.
• The expected firm value $P^*(\mu)$ is strictly increasing in $v^M_h, v^M_l$, and $k$ for $\mu < \mu^*$. It is constant and equal to $v^A_h$ for $\mu \geq \mu^*$.

• The threshold $\mu^*$ is strictly decreasing in $v^M_h, v^M_l$, and $k$.

The first two statements of the proposition suggest that, when the firm has better fundamentals $v^M$, the activist intervenes less and the manager exerts less effort in equilibrium. This is driving by the fact that the activist has greater incentive to refrain from intervention and pretend to have a successful firm with $v^M_h$. Moreover, a manager who is more readily motivated to exert effort ($k$ high) exerts more effort in equilibrium. However, this in turn allows the activist to intervene less. The last statement of the lemma suggests that despite the increasing incentive for the activist with high ability to bank on the threat of her activism and reduce intervention, the firm value is still increasing in $v^M$ and $k$ for $\mu < \mu^*$.

Since the activist also receives the expected firm value in expectation, this result points to the activist’s preference for identifying a potential target that maximises her expected profit from intervention. Denote by $\Pi^*(\mu) \equiv P^*(\mu) - v$ the expected profit of the activist with reputation $\mu$, where $v = v^M$ by Lemma 1.

**Proposition 3.** The expected profit of an activist $\Pi^*(\mu)$ is strictly decreasing in $v^M_l$. The expected profit is increasing in $v^M_h$ and $k$, and strictly increasing for $\mu < \mu^*$.

This result suggests that a firm is more likely to become a target of shareholder activism if the firm is currently in a bad position (low $v^M_l$), if the firm has high potential value (high $v^M_h$), and if the manager is readily motivated to exert effort (high $k$).

4 Portfolio Effects of Activists’ Short-Termism

This section studies the implications of an activist’s short-termism on her incentive to target multiple firms simultaneously. Specifically, we investigate whether holding multiple blocks weaken governance by spreading the activist too thinly, as commonly believed, and if not, which activist tends to have larger portfolios.

We extend the baseline model to allow the activist to establish blocks of size 1 at $t = 0$ in two ex ante identical firms. To capture the capacity constraint of the activist, we assume that, regardless
of the number of targets, it is common knowledge that she can only intervene in at most one firm within the time span considered. This could be due to, for example, limited staff resources.

The timeline with two target firms is similar to before. At \( t = 1 \), the manager of each portfolio firm choose unobservable effort \( e \), take as given the effort choice of the other firm \( e' \) and the activist’s optimal intervention strategy \( \{\lambda_h, \lambda_\ell\} \). At \( t = 2 \), the activist privately observes the payoff \( v^M_i \in \{v^M_h, v^M_\ell\} \), \( i \in \{1, 2\} \) of both firms, and \( v^A \in \{v^A_h, v^A_\ell\} \), the value of her alternative proposal. If at least one firm fails, the activist then decides whether to intervene in a failed firm given her private information \( v^A \). In particular, assume that the activist adopts a symmetric strategy towards the two target firms. That is, if both firms fail and the activist decides to intervene, each firm faces a \( \frac{1}{2} \) probability of receiving the intervention. Finally at \( t = 3 \), with independent probability \( \gamma \), true value of each firm is revealed and the activist exits her position in all firms at their respective market prices.

In this section we first characterize the equilibrium when the activist has two target firms, then consider how the choice of an activist to target one or more firms simultaneously depends on the activist’s ability \( \mu \).

### 4.1 Targeting Two Firms Simultaneously

In order to characterize the equilibrium with two targets, we again proceed backwards. At \( t = 3 \), each firm can be in one of three situations. If the activist intervened in firm \( i \), we henceforth refer to the other firm as \(-i\). Denote with \( P_i \) the expected value of the firm \( i \), and with \( P_{-i} \) the expected value of firm \(-i\). If the activist did not intervene, the two firms remain ex post identical from the market participant’s perspective, in the absence of additional information about the fundamental value of the firms. Denote with \( P_0 \) the expected value of each firm. Given an activist’s intervention strategy \( \{\lambda_h, \lambda_\ell\} \) and the managers’ effort level \( e \), the market prices when the true value of the firms are not revealed are described below\(^8\).

Following an intervention in firm \( i \), the price of firm \( i \), \( P_i \), depends only on the expected payoff of the activist’s strategy (Eq. \(13\)). This expression is identical to \( P_1 \) in the baseline case (Eq. \(1\)). For the other firm, since it does not receive activist intervention, the price \( P_{-i} \) reflects the expected

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\(^8\)As will become clear, the two firms’ managers choose the same level of effort in equilibrium, because the firms are ex ante identical.
result of the manager’s effort (Eq. 14). The updated probability of success in firm \( j \) given an intervention in firm \( i \) is given by \( \frac{e}{e + \frac{1}{2}(1-e)} > e \), because the fact that the activist intervenes with the firm \( i \) instead of firm \(-i\) contains favourable information about firm \(-i\)'s success.

\[
P_i \equiv \mathbb{E}[v_i | \text{Intervention in Firm } i] = Pr[v^A = v^A_h | I = 1](v^A_h - v^A_\ell) + v^A_\ell
\]

\[
P_{-i} \equiv \mathbb{E}[v_{-i} | \text{Intervention in Firm } i] = \frac{e}{e + \frac{1}{2}(1-e)} (v^M_h - v^M_{-i}) + v^M_{-i}
\]

If no intervention takes place, the market price is the same for both firms. It is equal to the expected value of the manager’s effort, conditional on no intervention in either firm.

\[
P_0 \equiv \mathbb{E}[v_i | I = 0]
\]

where \( \Lambda(\mu, \lambda_h, \lambda_\ell) \) is the conditional expected intervention probability in case of a failed manager, as given by Eq. 5. The absence of an intervention can be due to either both firms being successful or due to the activist’s strategic choice not to intervene. The price \( P_0 \) therefore reflects the expected probability that the manager of the firm is successful.

At \( t = 2 \), receives information \( v^M_i \) for both firms. If \( v^M_i = v^M_\ell \) for some \( i \in \{1, 2\} \), the activist decides upon her intervention strategy given her private information \( v^A \), to maximise her total expected payoff at exit:

\[
\lambda_h \equiv \text{arg max}_\lambda \lambda[\gamma(v^A_h + v^M_{-i}) + (1 - \gamma)(P_i + P_{-i})] + (1 - \lambda)[\gamma(v^M_h + v^M_{-i}) + (1 - \gamma)2P_0]
\]

\[
\lambda_\ell \equiv \text{arg max}_\lambda \lambda[\gamma(v^A_\ell + v^M_{-i}) + (1 - \gamma)(P_i + P_{-i})] + (1 - \lambda)[\gamma(v^M_\ell + v^M_{-i}) + (1 - \gamma)2P_0]
\]

These expressions are similar to Eq. 3 for the activist’s optimisation programme in the baseline model, plus the expected exit price for the second firm. We define as \( G(v^A, P_i, P_{-i}, P_0) \) the net
gain from a marginal intervention for the activist.

\[ G(v^A, P_i, P_{-i}, P_\emptyset) = \gamma(v^A - v^M_i) + (1 - \gamma)(P_i + P_{-i} - 2P_\emptyset) \quad (17) \]

It follows that the optimal intervention strategy for the activist, given her private information \( v^A \) and the market prices \( P_i, P_{-i} \) and \( P_\emptyset \), is such that she intervenes with certainty if \( G(\cdot) > 0 \), with zero probability if \( G(\cdot) < 0 \), and with any probability \( \lambda \in [0, 1] \) if \( G(\cdot) = 0 \).

At \( t = 1 \), each manager chooses an effort level \( e \) in anticipation of the activist’s intervention strategy \( \lambda(\mu, \lambda_h, \lambda_\ell) \) and the other manager’s effort level \( e' \) to minimise his costs:

\[ \min_e \left( ce^2 + e' + \frac{1}{2}(1 - e') \right) \Lambda(\mu, \lambda_h, \lambda_\ell, n) \quad (18) \]

By comparing with the manager’s optimisation programme in the baseline model Eq. 4, it is clear that having two targets simultaneously has a dilution effect on manager effort level. The presence of another firm reduces the condition probability of intervention in a failed firm by a factor of \( \left[ e' + \frac{1}{2}(1 - e') \right] < 1 \), which depends on the effort level of the other firm \( e' \). This is because, even when the manager fails, if the other firm also fails, the probability of receiving the intervention by the activist for each firm is \( \frac{1}{2} \). This also reduces the manager’s effort by the same factor, resulting in an optimal effort choice of \( e(e', \Lambda(\cdot)) = k \left[ e' + \frac{1}{2}(1 - e') \right] \Lambda(\mu, \lambda_h, \lambda_\ell) \).

The following proposition characterizes the equilibrium for an activist with two target firms in the limit as \( r \to 0 \). All equilibrium quantities are superscribed by **.

**Proposition 4.** As \( \gamma \to 0 \), there exists a threshold \( \mu^{**} \), such that the unique equilibrium is as follows:

- Equilibrium prices \( P_i^{**}(\mu), P_{-i}^{**}(\mu) \) and \( P_\emptyset^{**}(\mu) \) are characterised by Eq. 13–15 respectively;
- M’s equilibrium effort level is \( e^{**}(\mu) = \frac{\frac{1}{k}k^{**}(\mu)}{1 - \frac{1}{2}k^{**}(\mu)} \), where \( k^{**}(\mu) \equiv \Lambda(\mu, \lambda_h^{**}(\mu), \lambda_\ell^{**}(\mu)) \);
- A’s equilibrium intervention strategy \( \{\lambda_h^{**}(\mu), \lambda_\ell^{**}(\mu)\} \) are given by \( G(\mu, P_i^{**}(\cdot), P_{-i}^{**}(\cdot), P_\emptyset^{**}(\cdot)) = 0 \), where:
  - For \( \mu < \mu^{**} \), \( \lambda_h^{**}(\mu) = 1 \) and \( \lambda_\ell^{**}(\mu) \in (0, 1) \);
– For $\mu \geq \mu^\ast\ast$, $\lambda_h^\ast\ast(\mu) \in (0,1]$ and $\lambda_\ell^\ast\ast(\mu) = 0$;

- $\mu^\ast\ast$ is given by the solution to $v_h^A + P_{-i}(\mu) - 2P_0(\mu) = 0$, where $P_{-i}(\mu)$ and $P_0(\mu)$ are given by Eq. 14–15 respectively for $\lambda_h = 1$, $\lambda_\ell = 0$ and $e = \frac{1}{2}k\mu - 1$.

Proposition 4 suggests that the two-sided inefficiency is still present in the case with two target firms. The equilibrium is qualitatively similar to that with only one target firm. An activist with low reputation $\mu < \mu^\ast\ast$ intervenes excessively ($\lambda_h^\ast\ast(\mu) = 1$, $\lambda_\ell^\ast\ast(\mu) > 0$), whereas an activist with high reputation $\mu > \mu^\ast\ast$ foregoes sometimes value-enhancing intervention opportunity ($\lambda_h^\ast\ast(\mu) < 1$, $\lambda_\ell^\ast\ast(\mu) = 0$).

To highlight the effect of having more than one targets, in the following proposition we compare the equilibria with one and two targets.

**Proposition 5.** There exist $\bar{v}_h^A < v_h^M$ such that for $v_h^A < \bar{v}_h^A$,

- The threshold level of reputation is strictly higher when targeting two firms simultaneously, i.e. $\mu^\ast\ast > \mu^\ast$.

- Targeting two firms simultaneously, compared to targeting one firm, results in higher conditional intervention probability $\Lambda^\ast\ast(\mu) > \Lambda^*(\mu)$ for all $\mu$.

For $v_h^A \geq \bar{v}_h^A$, there exists $\mu'' \leq \mu^\ast\ast$ such that

- Targeting two firms simultaneously, compared to targeting one firm, results in higher conditional intervention probability $\Lambda^\ast\ast(\mu) > \Lambda^*(\mu)$ if and only if $\mu < \mu''$.

$\bar{v}_h^A$ is defined as $\bar{v}_h^A = \frac{k\mu'}{k\mu + (1-k\mu')(1-k\mu')} (v_h^M - v_\ell^M) + eM$, where $\mu'$ is given by the solution to $\lambda^\ast\ast(\mu) = \Lambda^*(\mu)$ for $v_h^A \geq \bar{v}_h^A$.

For $v_h^A$ not too close to $v_h^M$ (i.e. $v_h^A < \bar{v}_h^A$) or for $\mu$ not too high ($\mu < \mu''$), the interplay between the manager’s incentive to exert effort and the activist’s incentive to intervene efficiently leads to this interesting result. Because of the dilution effect of having multiple targets on the manager’s effort, each manager tends to exert less effort in equilibrium. In response, the activist intervenes with higher probability conditional on management failure, leading to higher $\Lambda^\ast\ast(\mu)$.

These properties have direct implications on the activist’s optimal number of targets, which we analyze in the following section.
4.2 Optimal Number of Targets

In this section we investigate the activist’s optimal number of targets by comparing her equilibrium profit over one or two targets, denoted with $\Pi_1(\mu)$ and $\Pi_2(\mu)$ respectively.

$$\Pi_1(\mu) \equiv e^*(\mu)(v^M_h - v^M_\ell) + [1 - e^*(\mu)] \left[ \mu \lambda^*_h(\mu)(v^A_h - v^M_h) + (1 - \mu) \lambda^*_\ell(\mu)(v^A_\ell - v^M_\ell) \right]$$  \hspace{1cm} (19)

$$\Pi_2(\mu) \equiv 2e^{**}(\mu)(v^M_h - v^M_\ell) + \left[1 - (e^{**}(\mu))^2\right] \left[ \mu \lambda^{**}_h(\mu)(v^A_h - v^M_h) + (1 - \mu) \lambda^{**}_\ell(\mu)(v^A_\ell - v^M_\ell) \right]$$  \hspace{1cm} (20)

The profit of the activist’s target consists of two terms. The first term is the direct value created by the manager’s effort, due to the threat of activism. When the activist targets two firms simultaneously, this term is multiplied by 2. The second term represents the value created through the direct intervention by the activist, which again has two components. An important component is the expected value of an activist’s intervention, conditional on a failed manager, represented by the last term in squared brackets in Eq. 19 and 20. We interpret this component as the efficiency of the activist’s direct intervention. The second component is the probability that at least one manager fails, which enables an activist to potentially invest (given by $[1 - e^*(\mu)]$ and $[1 - (e^{**}(\mu))^2]$ respectively when there is one or two targets).

We compare the profits of the activist with one and two target firms in order to study her optimal choice for each of the three components described above. Firstly, the total value created due to managers’ effort is strictly higher when the activist targets two firms simultaneously. That is, $2e^{**}(\mu) > e^*(\mu)$. This is driven by two opposing forces. On the one hand, the value created by each manager is strictly lower when the activist targets two firms simultaneously, due to the dilution to the threat of activism for each firm (Proposition 5). On the other hand, the effort exerted by the additional firm provides the activist with additional profit, leading to an overall positive effect on the total value creation due to managers’ effort.

The second and the most important effect of targeting multiple targets is on the efficiency of the activist’s direct intervention. While the first effect discussed above is unambiguously positive, this second effect depends on the activist’s ability. An additional target tends to improve the efficiency of the activist’s direct intervention only if the activist has high ability, and vice versa if the
activist has low ability. To see this, recall that when targeting multiple firms, the dilution effect of multiple targets on the manager’s effort forces the activist to intervene with higher probability in equilibrium by Proposition 5, i.e., $\Lambda^{**}(\mu) > \Lambda^*(\mu)$. However, the efficiency of this increased intervention probability depends on the activist’s ability. An activist with high ability intervenes insufficiently in equilibrium and foregoes sometimes value-enhancing intervention opportunity. Increased intervention probability thus increases the expected value of her intervention because her marginal intervention is value-enhancing. An activist with low ability, on the other hand, intervenes excessively in equilibrium even when she only has a value-destroying intervention opportunity. Increased intervention probability thus decreases the expected value of her intervention because her marginal intervention is value-destroying.

Lastly, the number of targets affects the probability that at least one manager fails and hence the probability of direct intervention through the effort exerted by the managers. With only one target, the activist intervenes only if the manager fails, with probability $[1 - e^*(\mu)]$; with two targets, the activist can intervene if at least one manager fails, with probability $[1 - (e^{**}(\mu))^2]$. This probability tends to be higher when the activist has two targets, because of the selection effect and because each manager tends to exert less effort when there are two targets.

The above analysis leads to the following result, which is further discussed below.

**Proposition 6.** There exists a threshold $\hat{\mu} \leq \mu^{**}$, such that $\Pi_1(\mu) > \Pi_2(\mu)$ if and only if $\mu < \hat{\mu}$, and vice versa. That is, an activist with low reputation $\mu < \hat{\mu}$ strictly prefers targeting only one firm, and an activist with high reputation $\mu > \hat{\mu}$ strictly prefers targeting two firms simultaneously. An activist with ability $\hat{\mu}$ is indifference between targeting one or two firms.

Since the first and the last effects strictly favor multiple targets, as discussed above, we focus on second effect on the efficiency of the activist’s direct intervention. An activist with high ability $\mu \geq \mu^{**} > \mu^*$ strictly prefers targeting more firms simultaneously. Targeting multiple firms allows the highly capable activist to effectively commit to more value-enhancing intervention, because the dilution of her threat of activism over each firm induces the managers to exert lower effort. An activist with lower ability, on the other hand, must trade off the value created by the manager of an additional target under the threat of activism, against the decreased efficiency of her direct intervention. That is, the dilution effect on the manager’s effort also decreases the efficiency of the
activist’s direct intervention since an activist with low ability intervenes excessively in equilibrium. As the effect from the threat of activism increases with the activist’s ability, the positive effect is more likely to dominate for an activist with higher ability.

It is for these reasons that an activist with higher ability can benefit from targeting multiple firms, even though she only has the capacity to engage in direct intervention in at most one firm, as illustrated in Fig 3. The following corollary summarizes the properties of the equilibrium when the activist targets the optimal number of firms.

Corollary 2. In the equilibrium in which the activist targets the optimal number of firms,

- The expected profit of an activist is increasing in her ability. It is strictly increasing for $\mu < \mu^*$ and for $\mu \in (\hat{\mu}, \mu^{**})$.

- The expected intervention probability conditional on management failure is increasing in the activist’s ability and discontinuous at $\hat{\mu}$.

- The effort level of the manager of a firm targeted by a activist is increasing in the activist’s ability for $\mu \in [0, \hat{\mu})$ and for $\mu \in (\hat{\mu}, 1]$. It is discontinuous at $\hat{\mu}$.

The results stated in Corollary 2 are illustrated in Fig. 4. The dashed line represents the equilibrium with only one target, and the thick solid line represents the equilibrium with two targets simultaneously. The results when the activist optimally chooses the number of targets is

\[ \hat{\mu} < \mu^* \] for some parameter values, such that $\mu^* \leq \mu^{**}$.
highlighted by the thick solid line. The left panel plots the equilibrium intervention probability of the activist conditional manager failure, and the right panel plots the equilibrium effort of the manager of a target firm. An activist with high ability optimally increases her number of targets, so as to dilute the threat of activism for each target firm, allowing the manager to exert lower effort. This effect is most clear for the an activist with critical ability level $\hat{\mu}$. In turn, the lower effort of the manager induces the activist to engage in more value-enhancing intervention ex post, improving her expected profit ex ante. For an activist with sufficiently high ability, the improved efficiency of her ex post intervention when targeting two firms simultaneously also implies greater effort by the managers of target firms, compared to when she only targets one firm.

5 Shareholder Short-Termism and Managerial Myopia

So far we have abstracted from any stock-based incentive compensation for the manager, and only consider the incentive effect of shareholder activism through a direct utility loss of $K$. In this section we extend the baseline model with one target firm, to consider the effect of activist shareholders’ short-termism, when the manager receives short- and long-term stock-based incentive compensation.

Assume that the manager receives $\delta \geq 0$ units of stock-based incentive compensation. Of the stock-based compensation, a fraction $\eta \in [0, 1]$ is long term and depends on the fundamental value
of the firm \( v \), and the remaining fraction \( (1 - \eta) \) is short term and depends on the market prices \( P \) in the absence of information regarding the fundamental value of the firm. The parameter \( \eta \) thus captures the horizon of the manager. This implies that the manager chooses an effort level \( e \) to maximise the value of his stock-based compensation, less the effort cost and the expected cost induced by potential activist intervention. Given the activist’s intervention strategy \((\lambda_h, \lambda_\ell)\), this can be expressed as

\[
\max_e \delta e [\eta v_h^M + (1 - \eta) P_0] \\
+ \delta (1 - e) \left[ \eta \left( \mu \lambda_h v_h^A + (1 - \mu) \lambda_\ell v_\ell^A + [1 - \Lambda(\mu, \lambda_h, \lambda_\ell)] v_\ell^M \right) \\
+ (1 - \eta) \Lambda(\mu, \lambda_h, \lambda_\ell) P_1 + [1 - \Lambda(\mu, \lambda_h, \lambda_\ell)] P_0 \right] \\
- c e^2 \frac{2}{2} - K(1 - e) \Lambda(\mu, \lambda_h, \lambda_\ell) (21)
\]

where \( \Lambda(\mu, \lambda_h, \lambda_\ell) \) is given by Eq. 5. The first line of Eq. 21 is the expected value of the manager’s stock-based compensation following success, the second line is the expected value of his stock-based compensation follows failure, taking into account the activist’s intervention strategy following managerial failure. Finally the last line of Eq. 21 are the costs, similar to Eq. 4 in the baseline model. It follows that the optimal effort choice of the manager is

\[
e(\mu, \lambda_h, \lambda_\ell, P_1, P_0) = k \Lambda(\mu, \lambda_h, \lambda_\ell) \\
+ \frac{\delta}{c} [v_h^M - v_\ell^M] - (\mu \lambda_h v_h^A - v_h^M) + (1 - \mu) \lambda_\ell (v_\ell^A - v_\ell^M)] \\
+ \frac{\delta}{c} (1 - \eta) \Lambda(\mu, \lambda_h, \lambda_\ell) (P_0 - P_1) (22)
\]

Clearly as \( \delta \to 0 \) the problem is identical to the baseline model. Eq. 22 illustrates that long- and short-term stock-based compensation have different effects on the manager’s incentive to exert effort, when facing an activist shareholder.

The second line of Eq. 22 represents the incentive effect of long-term stock-based compensation and is strictly positive. The term in the square bracket represents the marginal improvement in firm value due to managerial success. Due to activist intervention following managerial failure, the incentive to exert effort is weakened when the activist’s intervention efficiency improves, i.e. higher \( \mu, \lambda_h \) or lower \( \lambda_\ell \). Since an increase in the activist shareholders horizon \( \gamma \) improves her intervention
efficiency, as implied by Lemma 2, we have the following proposition:

**Proposition 7.** The effectiveness of long-term stock-based compensation on managerial incentive in equilibrium is decreasing in the activist shareholder’s horizon.

The third line of Eq. 22 represents the incentive effect of short-term stock-based compensation. Its sign depends on the prices $P_1$ and $P_0$ in equilibrium. Recall that Proposition 1 implies that the equilibrium prices are such that $(P_0^* - P_1^*) < 0$ for an activist with low ability, and vice versa for an activist with high ability (also see the right panel of Fig 2). We then have the following proposition.

**Proposition 8.** The effect of short-term stock-based compensation on managerial incentive in equilibrium is negative when facing an activist has low ability, but is positive when facing an activist with high ability. The effects are increasing in absolute values in the horizon of the activist.

This last result suggests that, activist shareholder has a perverse effect on the manager’s effort provision incentives when the manager has short-term stock-based compensation. Moreover, the effect is greater if the activist has longer horizon.

### 6 Conclusion

This paper investigates whether activist investors’ focus on short-term stock prices impedes their role in improving corporate governance. The model builds on the notion that both the act of intervention and the threat of an intervention can generate value for the target firm. We demonstrate that an activist intervenes excessively when her ability to conduct a value-enhancing intervention is low, but intervenes insufficiently if her ability is high. This is because the two channels are complements when the activist’s ability is low, but are substitutes otherwise. The effect of the activist’s threat on the manager’s effort and the expected firm value is increasing in the activist’s ability. Moreover, we show that an activist with higher ability finds it optimal to acquire stakes in multiple firms even when her capacity to conduct intervention is limited. While targeting multiple firms dilutes the effect of the activist’s threat on each manager, it improves a skilful activist’s profit by forcing her to substitute reduced managerial effort with more value-enhancing interventions in equilibrium.
7 Appendix

7.1 Proof of Proposition 1

The proof proceeds by first establishing that the equilibrium described in Proposition 1 exists, then showing that no other equilibrium exists.

The solutions to the activist’s and the manager’s optimization problems, given by Eq. 3 and 4 respectively, are as follows:

\[ \lambda_h(P_1, P_0) \begin{cases} 
    1, & \text{if } P_1 - P_0 > \frac{\gamma}{1 - \gamma} (v^M_v - v^A_h) \\
    0, & \text{if } P_1 - P_0 < \frac{\gamma}{1 - \gamma} (v^M_v - v^A_h) 
\end{cases} \quad (23) \]

\[ \lambda_e(P_1, P_0) \begin{cases} 
    1, & \text{if } P_1 - P_0 > \frac{\gamma}{1 - \gamma} (v^M_v - v^A_e) \\
    0, & \text{if } P_1 - P_0 < \frac{\gamma}{1 - \gamma} (v^M_v - v^A_e) 
\end{cases} \quad (24) \]

\[ e(\lambda(\mu, \lambda_h, \lambda_e)) = k\lambda(\mu, \lambda_h, \lambda_e) \quad (25) \]

We now first show that the equilibrium is as described for \( \mu \) in each of the 3 regions. It is straightforward to verify that the equilibrium for \( \mu \in [\underline{\mu}, \overline{\mu}] \) described in Proposition 1 is unique. The equilibrium prices in this region are such that \( P^*_1(\mu) - P^*_0(\mu) \in [-\frac{\gamma}{1 - \gamma} (v^A_h - v^M_v), \frac{\gamma}{1 - \gamma} (v^M_v - v^A_e)] \).

Denote the prices given in Eq. 1 and 2 by \( P_1(\mu, \lambda_h, \lambda_e) \) and \( P_0(\mu, \lambda_h, \lambda_e) \) respectively, which take into account rational expectation regarding \( M \’s effort choice e(\Lambda(\mu, \lambda_h, \lambda_e)) \). Notice that the threshold \( P_0^*(\mu) \) is given by \( P_0^*(\mu) = P_0(\mu, 1, 0) \).

For \( \mu < \underline{\mu} \), \( P_1(\mu, 1, 0) - P_0(\mu, 1, 0) > \frac{\gamma}{1 - \gamma} (v^M_v - v^A_e) \). Notice that \( P_1(\mu, \lambda_h, 1) < P_0(\mu, \lambda_h, 1) \) for all \( \mu \) and \( \lambda_h \), and that \( P_1(\mu, 1, \lambda_e) \) is decreasing in \( \lambda_e \) whereas \( P_0(\mu, 1, \lambda_e) \) is increasing in \( \lambda_e \). It then follows that there exists a unique \( \lambda^*_e \in (0, 1) \) such that \( P_1(\mu, 1, \lambda^*_e) - P_0(\mu, 1, \lambda^*_e) = \frac{\gamma}{1 - \gamma} (v^A_h - v^M_v) \). The equilibrium for \( \mu < \underline{\mu} \) is as described in Proposition 1.

Following similar reasoning, for \( \mu > \overline{\mu} \), \( P_0(\mu, 1, 0) - P_1(\mu, 1, 0) > \frac{\gamma}{1 - \gamma} (v^A_h - v^M_v) \). Since \( e(\Lambda(\mu, 0, 0)) = 0 \), \( P_1(\mu, 0, 0) > P_0(\mu, 0, 0) \) for all \( \mu \). Further, \( P_1(\mu, \lambda_h, 0) \) is constant whereas
$P_0(\mu, \lambda_h, 0)$ is increasing in $\lambda_h$. Therefore there exists a unique $\lambda_h^* \in (0, 1)$ such that $P_0(\mu, \lambda_h^*, 0) - P_1(\mu, \lambda_h^*, 0) > \frac{\gamma}{1-\gamma}(v_h^M - v_{\ell}^A)$. The equilibrium for $\mu > \bar{\mu}$ is as described in Proposition I.

We now proceed to show by contradiction that no other equilibrium exists. Suppose there exists an equilibrium in which $P_1(\cdot) - P_0(\cdot) > \frac{\gamma}{1-\gamma}(v_h^M - v_{\ell}^A)$. Then it must be that $\lambda_\ell = \lambda_h = 1$. However, since $P_1(\mu, 1, 1) < P_0(\mu, 1, 1)$ for all $\mu$, this is a contradiction. Alternatively, suppose there exists an equilibrium in which $P_0(\cdot) - P_1(\cdot) > \frac{\gamma}{1-\gamma}(v_{\ell}^A - v_h^M)$. Then it must be that $\lambda_\ell = \lambda_h = 0$ which also yields a contradiction since $P_1(\mu, 0, 0) > P_0(\mu, 0, 0)$ for all $\mu$.

### 7.2 Proof of Corollary I

$A$’s intervention strategy when $v^A = v_{\ell}^A$ is $\lambda_h^*(\mu) = 1$ for $\mu \leq \bar{\mu}$. For $\mu > \bar{\mu}$, $\lambda_h^*(\mu)$ is given by $P_0(\mu, \lambda_h, 0) - v_{\ell}^A = \frac{\gamma}{1-\gamma}(v_h^A - v_{\ell}^A)$. Since $P_0(\mu, \lambda_h, 0)$ is increasing in both $\mu$ and $\lambda_h$, $\lambda_h^*(\mu)$ is decreasing in $\mu$.

$A$’s intervention strategy for $v^A = v_{\ell}^A$ is $\lambda_\ell^*(\mu) = 0$ for $\mu \geq \underline{\mu}$. For $\mu < \underline{\mu}$, $\lambda_\ell^*(\mu)$ is given by $P_1(\mu, 1, \lambda_\ell) - P_0(\mu, 1, \lambda_\ell) = \frac{\gamma}{1-\gamma}(v_{\ell}^M - v_{\ell}^A)$. $P_1(\mu, 1, \lambda_\ell) - P_0(\mu, 1, \lambda_\ell)$ is increasing in $\lambda_\ell$ but can be non-monotonic in $\mu$ because both $P_1(\cdot)$ and $P_0(\cdot)$ are increasing in $\mu$. Therefore $\lambda_\ell^*(\mu)$ is strictly positive but non-monotonic for $\mu < \underline{\mu}$.

We now turn to the expected intervention probability conditional on management failure $\Lambda^*(\mu)$. First consider the cases for $\mu \geq \underline{\mu}$. Rewrite $P_0(\mu, \lambda_h, \lambda_\ell)$ as $P_0(\Lambda(\mu, \lambda_h, \lambda_\ell))$, given by.

$$P_0(\Lambda) \equiv \frac{e(\Lambda)}{e(\Lambda) + [1 - e(\Lambda)](1 - \Lambda)(v_h^M - v_{\ell}^M) + v_{\ell}^M} \quad (26)$$

Therefore $P_0^*(\mu) = P_0(\Lambda^*(\mu))$. For $\mu \geq \bar{\mu}$, $P_0^*(\Lambda^*(\mu)) = v_{\ell}^A + \frac{\gamma}{1-\gamma}(v_h^A - v_{\ell}^M)$. Since $P_0(\Lambda)$ is strictly increasing in $\Lambda$, $\Lambda^*(\mu)$ must be constant for all $\mu > \bar{\mu}$ and equal to $\bar{\mu}$. For $\mu \in [\underline{\mu}, \bar{\mu}]$, $\Lambda^*(\mu) = \mu$ as $\lambda_h^* = 1$ and $\lambda_\ell^* = 0$. Therefore $\Lambda^*(\mu)$ is strictly increasing in $\mu$ for $\mu \geq \underline{\mu}$.

We proceed to show that $\Lambda^*(\mu)$ is strictly increasing for $\mu < \underline{\mu}$ by contradiction. Rewrite $P_1(\mu, \lambda_h, \lambda_\ell)$ as $P_1(\mu, \Lambda(\mu, \lambda_h, \lambda_\ell))$, given by.

$$P_1(\mu, \Lambda) \equiv \min\{1, \frac{\mu}{\Lambda}\}(v_h^A - v_{\ell}^A) + v_{\ell}^A \quad (27)$$

The equilibrium price $P_1^*(\mu)$ can then be expressed as $P_1(\mu, \Lambda^*(\mu))$. Notice that $P_1(\mu, \Lambda^*(\cdot))$ is
strictly increasing in $\mu$ and strictly decreasing in $\Lambda$ for $\mu < \underline{\mu}$. Suppose that $\Lambda^*(\mu)$ is weakly decreasing in $\mu$. Then there exist $\mu', \mu'' < \underline{\mu}$, such that $\mu' > \mu''$ and $\Lambda^*(\mu') \leq \Lambda^*(\mu'')$. This implies that $P^*_1(\mu', \Lambda^*(\mu')) > P^*_1(\mu'', \Lambda^*(\mu''))$ and $P^*_0(\Lambda^*(\mu)) < P^*_0(\Lambda^*(\mu''))$. This contradicts with the equilibrium condition that $P^*_1(\mu) - P^*_1(\mu) = \frac{\gamma}{1-\gamma} (v^M_\ell - v^A_\ell)$ for all $\mu < \underline{\mu}$.

That $e^*(\mu)$ is weakly increasing in $\mu$ and strictly increasing for $\mu \leq \bar{\mu}$ follows immediately from the property of $\Lambda^*(\mu)$, since $e^*(\mu) = e(\Lambda^*(\mu))$, where $e(\Lambda)$ is strictly increasing in $\Lambda$.

For the properties of the equilibrium prices $P^*_1(\mu)$ and $P^*_0(\mu)$, first consider the cases for $\mu \geq \underline{\mu}$. $P^*_1(\mu) = v^A_\ell$ for all $\mu \geq \underline{\mu}$. The property of $\Lambda^*(\mu)$ implies that $P^*_0(\mu) = P_0(\Lambda^*(\mu))$ is strictly increasing for $\mu \in [\underline{\mu}, \bar{\mu}]$, and constant and equal to $P^*_0(\bar{\mu})$ for $\mu > \bar{\mu}$.

For $\mu < \underline{\mu}$, the equilibrium intervention strategy $\lambda^*_\ell(\mu)$ and prices are given by $P_1(\mu, 1, \lambda_\ell) - P_0(\mu, 1, \lambda_\ell) = \frac{\gamma}{1-\gamma} (v^M_\ell - v^A_\ell)$. Implicitly differentiating this equality suggests that $\frac{\partial \lambda^*_\ell(\mu)}{\partial \mu} = -\frac{\partial P^*_1(\cdot)}{\partial \mu} - \frac{\partial P^*_0(\cdot)}{\partial \mu}$. The derivative of the equilibrium price $P^*_1(\mu)$ with regard to $\mu$ is then given by

$$\frac{\partial P^*_1(\mu)}{\partial \mu} = \frac{\partial P^*_1(\mu, 1, \lambda^*_\ell(\mu))}{\partial \mu} + \frac{\partial P^*_1(\mu, 1, \lambda^*_\ell(\mu))}{\partial \lambda^*_\ell(\mu)} \frac{\partial \lambda^*_\ell(\mu)}{\partial \mu}$$

$$= \frac{\partial P^*_1(\cdot)}{\partial \mu} - \beta(\mu, \lambda^*_\ell(\mu)) \left[ \frac{\partial P^*_1(\cdot)}{\partial \mu} - \frac{\partial P^*_0(\cdot)}{\partial \mu} \right]$$

$$= \left[ 1 - \beta(\mu, \lambda^*_\ell(\mu)) \right] \frac{\partial P^*_1(\cdot)}{\partial \mu} + \beta(\mu, \lambda^*_\ell(\mu)) \frac{\partial P^*_0(\cdot)}{\partial \mu} > 0$$

where $\beta(\mu, \lambda) = \frac{\partial P^*_1(\mu, 1, \lambda)}{\partial \lambda} - \frac{\partial P^*_0(\mu)}{\partial \lambda} \in (0, 1)$ for $\lambda \in (0, 1)$. Since $P^*_1(\mu) - P^*_0(\mu) = \frac{\gamma}{1-\gamma} (v^M_\ell - v^A_\ell)$, $P^*_0(\mu)$ is also strictly increasing in $\mu$ for $\mu < \underline{\mu}$.

7.3 Proof of Lemma

This result follows immediately from Proposition.

7.4 Proof of Proposition

We first examine the effect of firm characteristics on the threshold $\mu^*$, and then study the effect of firm characteristics on $\Lambda^*(\mu)$, $e^*(\mu)$ and $P^*(\mu)$ respectively.

The threshold $\mu^*$ is given by $v^A_\ell - P^*_0(\mu) = 0$, where $P^*_0(\mu)$ is stated in Proposition. Partially differentiating $P^*_0(\cdot)$ suggests that it is strictly increasing in $\mu$, $v^M_\ell$, $v^M_e$ and $k$. This implies that $\mu^*$ is strictly decreasing in $v^M_\ell$, $v^M_e$ and $k$. 29
We now proceed to examine the effect of firm characteristics on \( \Lambda^\ast(\cdot) \). Recall that \( \Lambda^\ast(\cdot) \) can be characterised by the solution to \( P_1(\mu, \Lambda(\cdot)) - P_0(\Lambda(\cdot)) = 0 \). \( P_1(\mu, \Lambda^\ast(\cdot)) \) is strictly decreasing in \( \Lambda \) for \( \mu < \mu^\ast \) and independent of \( \Lambda \) otherwise, and independent of \( v_h^M, v_\ell^M \) and \( k \). \( P_0(\cdot) \) is strictly increasing in \( \Lambda, v_h^M, v_\ell^M \) and \( k \). This implies that \( \Lambda^\ast(\cdot) \) is strictly decreasing in \( v_h^M, v_\ell^M \) and \( k \) \( \forall \mu \in [0, 1] \).

The manager’s effort in equilibrium \( e^\ast(\cdot) \) is strictly decreasing in \( v_h^M \) and \( v_\ell^M \) follows immediately from the properties of \( \Lambda^\ast(\cdot) \) because \( e^\ast(\cdot) = k\Lambda^\ast(\cdot) \). To study the effect of \( k \) on manager’s effort in equilibrium, rewrite \( P_1(\mu, \lambda_h, \lambda_\ell) \) as \( P_1(\mu, e(\mu, \lambda_h, \lambda_\ell)) \equiv \min\{1, \frac{\mu}{e(\mu, \lambda_h, \lambda_\ell)/k}\}(v_h^A - v_\ell^A) + v_\ell^A \), and \( P_0(\mu, \lambda_h, \lambda_\ell) \) as \( P_0(e(\mu, \lambda_h, \lambda_\ell)) \equiv \frac{e(\mu, \lambda_h, \lambda_\ell)}{e(\mu, \lambda_h, \lambda_\ell) + [1 - e(\mu, \lambda_h, \lambda_\ell)]|1 - e(\mu, \lambda_h, \lambda_\ell)/k|}(v_h^M - v_\ell^M) + v_\ell^M \). Therefore the equilibrium effort level \( e^\ast(\mu) \) can be characterised by the solution to \( P_1(\mu, e^\ast(\mu)) - P_0(e^\ast(\mu)) = 0 \). \( P_1(\cdot) \) is strictly decreasing in \( e \) and strictly increasing in \( k \) for \( \mu < \mu^\ast \) and independent otherwise. \( P_0(\cdot) \) is strictly increasing in \( e \) and strictly decreasing in \( k \). This implies that the equilibrium effort level \( e^\ast(\mu) \) is strictly increasing in \( k \).

Finally we consider the effect of firm characteristics on equilibrium prices. For \( \mu < \mu^\ast(\cdot) \). Since the equilibrium price \( P^\ast(\mu) = p_1^\ast(\mu) = \frac{\mu}{\Lambda^\ast(\mu)}(v_h^A - v_\ell^A) + v_\ell^A \), it follows from the properties of \( \Lambda^\ast(\cdot) \) that the equilibrium price is strictly increasing in \( v_h^M \), \( v_\ell^M \) and \( k \) for \( \mu < \mu^\ast \). For \( \mu \geq \mu^\ast \), the equilibrium price is constant and equal to \( P^\ast(\mu) = v_h^A \), as shown in Corollary 1.

### 7.5 Proof of Proposition 3

We prove this proposition by considering the effect of a change in \( v_h^M \), \( v_\ell^M \) and \( k \) respectively.

First, we consider the effect of a change in \( v_\ell^M \). As \( P^\ast(\mu) \) is equal to \( P_0 \) in equilibrium, we rewrite \( \Pi^\ast(\mu) = P^\ast(\mu) - v_\ell^M \) as following

\[
\Pi^\ast(\mu) = \frac{e^\ast(\mu)}{e^\ast(\mu) + [1 - e^\ast(\mu)]|1 - e^\ast(\mu)/k|}(v_h^M - v_\ell^M) \tag{28}
\]

Since the first term is strictly increasing in \( e^\ast(\mu) \), which is strictly decreasing in \( v_\ell^M \) by Proposition 2 and the second term \( (v_h^M - v_\ell^M) \) is strictly decreasing in \( v_\ell^M \), the overall expression for \( \Pi^\ast(\mu) \) is strictly decreasing in \( v_\ell^M \).

Next, we consider the effect of a change in \( v_h^M \) and a change in \( k \). As \( P^\ast(\mu) \) is also equal to \( P_1 \)
in equilibrium, we rewrite \( \Pi^*(\mu) = P^*(\mu) - v^M_\ell \) as following

\[
\Pi^*(\mu) = \begin{cases} 
\frac{\mu}{\Lambda^*(\mu)}(v^A_h - v^A_\ell) + v^A_\ell - v^M_\ell, & \text{if } \mu < \mu^* \\
v^A_h - v^M_\ell, & \text{otherwise}
\end{cases}
\]

Since \( \Lambda^*(\mu) \) is strictly decreasing in \( v^M_h \) and \( k \) by Proposition 2, it follows that \( P^*_i(\mu) \) is strictly increasing in \( v^M_h \) and \( k \) for \( \mu < \mu^* \). It is constant and equal to \( v^A_h - v^M_\ell \) for \( \mu \geq \mu^* \), independent of \( v^M_h \) and \( k \).

### 7.6 Proof of Proposition 4

The proof follows closely the proof of Proposition 1. We first characterize the equilibrium for any \( \gamma \in [0, 1] \), then finally take the limit as \( \gamma \to 0 \).

The solution to the activist’s optimization problem, given by Eq. 16 is as follows

\[
\lambda^*_h(P_i, P_{-i}, P_{\emptyset}) = \begin{cases} 
1, & \text{if } G(v^A_h, P_i, P_{-i}, P_{\emptyset}) > 0 \\
0, & \text{if } G(v^A_h, P_i, P_{-i}, P_{\emptyset}) < 0
\end{cases}
\]  

\[
\lambda^*_\ell(P_i, P_{-i}, P_{\emptyset}) = \begin{cases} 
1, & \text{if } G(v^A_\ell, P_i, P_{-i}, P_{\emptyset}) > 0 \\
0, & \text{if } G(v^A_\ell, P_i, P_{-i}, P_{\emptyset}) < 0
\end{cases}
\]  

Since \( G(v^A_h, P_i, P_{-i}, P_{\emptyset}) > G(v^A_\ell, P_i, P_{-i}, P_{\emptyset}) \) for all \( P_i, P_{-i} \) and \( P_{\emptyset} \), the equilibrium intervention strategy must be either \( \lambda^*_h = 1 \) and \( \lambda^*_\ell \in [0, 1] \), or \( \lambda^*_h \in [0, 1] \) and \( \lambda^*_\ell = 0 \).

The solution to the manager’s optimization problem, given by Eq. 18 take as given the other manager’s effort choice \( e' \) and the activist’s intervention strategy \( \Lambda(\mu, \lambda^*_h, \lambda^*_\ell) = \mu \lambda^*_h + (1 - \mu) \lambda^*_\ell \) is given by \( e(e', \Lambda(\cdot)) = k[e' + \frac{1}{2}(1 - e')]\Lambda(\mu, \lambda^*_h, \lambda^*_\ell) \). Since the other manager faces a similar optimization problem, his optimal effort choice is given by \( e'(e, \Lambda(\cdot)) = k[e + \frac{1}{2}(1 - e)]\Lambda(\mu, \lambda^*_h, \lambda^*_\ell) \).

Solving both equations simultaneous yields the following optimal effort choice for each manager,
given the activist’s intervention strategy

\[ e(\Lambda(\cdot)) = \frac{\frac{1}{2}k\Lambda(\cdot)}{1 - \frac{1}{2}k\Lambda(\cdot)} \]  

(31)

We proceed to show that, similar to the case with only one target, there exists thresholds \( \mu \) and \( \mu \), such that the equilibrium intervention strategy is \( \lambda_h^*(\mu) = 1 \) and \( \lambda_\ell^*(\mu) > 0 \) for \( \mu < \mu \), it is \( \lambda_h^*(\mu) = 1 \) and \( \lambda_\ell^*(\mu) = 0 \) for \( \mu \in [\mu, \mu] \), and it is \( \lambda_h^*(\mu) < 1 \) and \( \lambda_\ell^*(\mu) = 0 \) for \( \mu > \mu \). The thresholds \( \mu \) and \( \mu \) are given implicitly by the solutions to \( v_h^A + P_{-i}(\mu) - 2P_{0}^\ell(\mu) = 0 \) and \( v_\ell^A + P_{-i}(\mu) - 2P_{0}^\ell(\mu) = 0 \) respectively, where \( P_{-i}(\mu) \) and \( P_{0}^\ell(\mu) \) are given by Eq. 14-15 respectively, for \( \lambda_h = 1 \), \( \lambda_\ell = 0 \) and \( e = \frac{\frac{1}{2}k\mu}{1 - \frac{1}{2}k\mu} \).

Since the equilibrium prices \( P_i, P_{-i} \) and \( P_{0} \) are determined by the activist’s intervention strategy \( (\lambda_h, \lambda_\ell) \) and the manager’s effort \( e(\Lambda(\mu, \lambda_h, \lambda_\ell)) \) given by Eq. 31, we can rewrite \( G(v^A, P_i, P_{-i}, P_{0}) \) as \( G(v^A, \mu, \lambda_h, \lambda_\ell) \), defined as follows.

\[ G(v^A, \mu, \lambda_h, \lambda_\ell) \equiv \gamma(v^A - v_\ell^M) + (1 - \gamma)[P_i(\mu, \lambda_h, \lambda_\ell) - P_{#}(\mu, \lambda_h, \lambda_\ell)] \]  

(32)

where \( P_{#}(\mu, \lambda_h, \lambda_\ell) \equiv 2P_{0} - P_{-i} \). This allows us to draw an analogy between this case with two targets, and the baseline case with only one target. \( P_i - P_{#} \) in this case is similar to \( P_i - P_{0} \) in the baseline case. The thresholds \( \mu \) and \( \mu \) can now also be characterized as \( G(v^A, \mu, 1, 0) = 0 \) and \( G(v_h^A, \mu, 1, 0) = 0 \) respectively. We next verify that the equilibrium is indeed as described above and is unique.

Firstly, we verify that, for \( \mu \in [\mu, \mu] \), the unique equilibrium is \( \lambda_h^*(\mu) = 1 \) and \( \lambda_\ell^*(\mu) = 0 \). To see this, notice that \( G(v^A, \mu, 1, 0) \) is strictly decreasing in \( \mu \), because \( P_i(\mu, 1, 0) \) is strictly decreasing in \( \mu \) and \( P_{#}(\mu, 1, 0) \) is strictly increasing in \( \mu \). This implies that \( G(v^A, \mu, 1, 0) > 0 \) and \( G(v_A, \mu, 1, 0) < 0 \) for all \( \mu \in [\mu, \mu] \), and \( \lambda_h^*(\mu) = 1 \) and \( \lambda_\ell^*(\mu) = 0 \) is indeed the equilibrium strategy.

Next, consider \( \mu < \mu \). In this case, \( G(v_h^A, \mu, 1, 0) > G(v_\ell^A, \mu, 1, 0) > 0 \), implying that the equilibrium strategy is such that \( \lambda_h = 1 \) and \( \lambda_\ell > 0 \). Notice that \( G(v_A, \mu, 1, 1) < 0 \) for all \( \mu \), as \( P_i(\mu, \lambda_h, 1) \leq v_h^A < P_{#}(\mu, 1, 1) = v_\ell^M \). Furthermore, \( P_i(\mu, 1, \lambda_\ell) \) is strictly decreasing in \( \lambda_\ell \) whereas \( P_{#}(\mu, 1, \lambda_\ell) \) is strictly increasing in \( \lambda_\ell \) for all \( P_{#}(\cdot) \leq v_\ell^M \). By continuity, there exists a unique \( \lambda_\ell^* \in (0, 1) \) such that \( G(v_\ell^A, \mu, 1, \lambda_\ell^*) = 0 \), which is the equilibrium intervention strategy.
Following similar reasoning, consider the case $\mu > \bar{\mu}$. In this case, $G(v^A_\ell, \mu, 1, 0) < G(v^A_h, \mu, 1, 0) < 0$, implying that the equilibrium strategy is such that $\lambda_h < 1$ and $\lambda_\ell = 0$. Notice that $G(v^A_h, \mu, 0, 0) > 0$ for all $\mu$, as $P_i(\mu, 0, 0) > P_\#(\mu, 0, 0) = 0$. Furthermore, $P_i(\mu, \lambda_h, 0)$ constant and equal to $v^A_h$ whereas $P_\#(\mu, \lambda_h, 0)$ is strictly decreasing in $\lambda_h$ for all $P_\#(\cdot) \leq v^M_h$. By continuity, there exists a unique $\lambda^{**} \in (0, 1)$, such that $G(v^A_h, \mu, \lambda^{**}, 0) = 0$, which is the equilibrium intervention strategy.

Having characterized the equilibrium for the general case with $\gamma \in [0, 1]$, it is now straightforward to see that, as $\gamma \to 0$, $\mu \to \bar{\mu}$ because $G(v^A, P_\ell, P_{-i}, P_0)$ becomes independent of $v^A$. Denote this threshold $\mu^{**}$.

### 7.7 Proof of Proposition 5

We first compare the thresholds for the case with one and two targets, $\mu^*$ and $\mu^{**}$. Recall that $\mu^*$ is given by $P_1(\mu^*, 1, 0) - P_0(\mu^*, 1, 0) = 0$, whereas $\mu^{**}$ is given by $P_1(\mu^{**}, 1, 0) - P_\#(\mu^{**}, 1, 0) = 0$. Notice that $P_1(\mu, \lambda_h, \lambda_\ell) = P_1(\mu, \lambda_h, \lambda_\ell)$ for all $\mu$, $\lambda_h$ and $\lambda_\ell$. This is because both expressions represent the expected value of an activist’s intervention, conditional on the intervention and for a given intervention strategy $(\lambda_h, \lambda_\ell)$. Moreover, it can be shown that $P_\#(\mu, 1, 0) < P_0(\mu, 1, 0)$ if and only if $\mu < \mu'$, where $P_\#(\mu', 1, 0) = P_0(\mu', 1, 0) = \bar{v}^A_h$. This then implies that $\mu^* < \mu^{**}$ if and only if $v^A_h < \bar{v}^A_h$, as both $P_\#(\mu, 1, 0)$ and $P_0(\mu, 1, 0)$ are strictly increasing in $\mu$.

We next consider the equilibrium intervention probability of the activist conditional on at least one firm failing. Consider first $\mu \leq \min\{\mu^*, \mu^{**}\}$. In this case, recall that $\Lambda^*(\mu)$ for $\mu \leq \mu^*$ can be characterized by the solution to $P_0(\mu, \Lambda) = P_1(\Lambda)$, where $P_0(\mu, \Lambda)$ and $P_1(\Lambda)$ are given by Eq. 26 and 27 respectively. Similarly, $\Lambda^{**}(\mu)$ can be characterized by the solution to $P_\#(\mu, \Lambda) = P_1(\Lambda)$, where $P_1(\mu, \Lambda) = P_1(\mu, \Lambda)$ for the same reason as before, and $P_\#(\Lambda)$ is given by

$$P_\#(\Lambda) = 2P_0 - P_{-i} \equiv \frac{(k\Lambda)^2(2 - k\Lambda)}{4 - 4(1 + k)\Lambda + k(4 + k)\Lambda^2}(v^M_h - v^M_\ell) + v^M_\ell$$  \hspace{1cm} (33)

It can be shown that $P_\#(\Lambda) < P_0(\Lambda)$ if and only if $\Lambda < \mu'$, because $P_\#(\Lambda)$ and $P_0(\Lambda)$ have the same functional form as $P_\#(\mu, 1, 0)$ and $P_0(\mu, 1, 0)$, respectively.

The results then follows from the monotonicity of $\Lambda^*(\mu)$ and $\Lambda^{**}(\mu)$. If $v^A_h = \bar{v}^A_h$, $\mu^* = \mu^{**} = \mu'$. In this case $\Lambda^*(\mu) < \Lambda^{**}(\mu)$ for all $\mu < \mu'$ and $\Lambda^*(\mu) = \Lambda^{**}(\mu) = \mu'$ for all $\mu \geq \mu'$. If $v^A_h < \bar{v}^A_h$, $\mu^* < \mu^{**} < \mu'$. In this case, $\Lambda^*(\mu) < \Lambda^{**}(\mu)$ for all $\mu$. Finally, if $v^A_h > \bar{v}^A_h$, $\mu^* > \mu^{**} > \mu'$. In this
case, there exists $\mu'' < \mu'$ such that $\Lambda^*(\mu'') = \Lambda^{**}(\mu'') = \mu'$. $\Lambda^*(\mu) < \Lambda^{**}(\mu)$ if and only if $\mu < \mu''$.

7.8 Proof of Proposition 6

In order to prove this proposition, we first show that, for $\mu \geq \mu^{**}$, $\Pi_1(\mu) < \Pi_2(\mu)$. We establish the result for $v^A_h < \tilde{v}^A_h$ and the complementary case separately. Finally we consider the case of $\mu < \mu^{**}$.

Suppose $v^A_h < \tilde{v}^A_h$, we then have $\mu^* < \mu^{**}$ and $\Lambda^*(\mu) < \Lambda^{**}(\mu)$ for all $\mu$. Consider $\mu \geq \mu^{**} > \mu^*$. In this case, the profits with one and two targets are equal to $\Pi_1(\mu^*)$ and $\Pi_2(\mu^{**})$, respectively, for all $\mu \geq \mu^{**}$.

$$\Pi_1(\mu^*) = k\mu^*(v^M_h - v^M_\ell) + (1 - k\mu^*)\mu^*(v^A_h - v^M_\ell)$$

(34)

$$\Pi_2(\mu^{**}) = \frac{k\mu^{**}}{1 - \frac{k^{**}}{2k\mu^{**}}(v^M_h - v^M_\ell)} + \left[1 - \left(\frac{k\mu^{**}}{1 - \frac{k^{**}}{2k\mu^{**}}}\right)^2\right]^{\mu^{**}}(v^A_h - v^M_\ell)$$

(35)

It can be shown that the right hand side of Eq. (34) is smaller than the right hand side of Eq. (35) if $\mu^* = \mu^{**}$. Since the right hand side of Eq. (35) is increasing in $\mu^{**}$, it then follows from $\mu^* < \mu^{**}$ that $\Pi_1(\mu^*) < \Pi_2(\mu^{**})$.

Suppose now that $v^A_h \geq \tilde{v}^A_h$. In this case $\mu^* > \mu^{**} > \mu'$. For $\mu \geq \mu^{**}$, we have $P_\#(\Lambda^*(\mu)) > P_0(\Lambda^*(\mu))$ as shown in Appendix 7.7. In equilibrium, since $P_1(\mu, \Lambda^*(\mu)) = P_0(\Lambda^*(\mu))$, we have $\Pi_1(\mu) = P_1(\Lambda^*(\mu)) - v^M_\ell$. Similarly, since $P_\#(\Lambda^{**}(\mu)) = P_1(\mu, \Lambda^{**}(\mu))$, we have $\Pi_2(\mu) = 2[P_0(\cdot) - v^M_\ell]$ in equilibrium. For $\mu \geq \mu^{**} > \mu'$, $\Lambda^*(\mu) > \Lambda^{**}(\mu)$, which implies that $P_\#(\Lambda^{**}(\mu)) = P_1(\Lambda^{**}(\mu)) > P_1(\Lambda^*(\mu))$. This further implies that $\Pi_2(\mu) = 2[P_0(\cdot) - v^M_\ell] > P_1(\Lambda^*(\mu)) - v^M_\ell = \Pi_1(\mu)$, given that $P_{-1}(\cdot) \geq v^M_\ell$.

Consider finally $\mu < \mu^{**}$. The profits with one and two targets are given by Eq. (19) and (20) respectively. It can be shown that, for $\mu < \min\{\mu^*, \mu^{**}\}$, there exists at most one $\hat{\mu}$, such that $\Pi_1(\hat{\mu}) = \Pi_2(\hat{\mu})$.

If $\Pi_2(\mu^*) \leq \Pi_1(\mu^*) = v^A_h$, then by monotonicity and continuity, there exists $\hat{\mu} \in [\mu^*, \mu^{**}]$ such that $\Pi_1(\mu) < \Pi_2(\mu)$ if and only if $\mu > \hat{\mu}$. If $\Pi_2(\mu^*) > \Pi_1(\mu^*)$, there can be two cases. If $\frac{\partial \Pi_1(\mu)}{\partial \mu} |_{\mu \to 0} < \frac{\partial \Pi_2(\mu)}{\partial \mu} |_{\mu \to 0}$, monotonicity and continuity implies that $\Pi_1(\mu) < \Pi_2(\mu)$ for all $\mu$. That is, $\hat{\mu} = 0$. Otherwise, there exists $\hat{\mu} \in (0, \mu^*)$ such that $\Pi_1(\mu) < \Pi_2(\mu)$ if and only if $\mu > \hat{\mu}$.

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7.9  Proof of Corollary 2

This Corollary follows immediately from Proposition 5 and 6.
References


